

# 求解涡扇发动机数学模型 的有限域搜索方法

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**【摘要】** 提出一种基于 N-R 迭代方法的有限域优化搜索法, 用于求解涡扇发动机数学模型。该方法不仅具有 N-R 优化迭代求解特色, 还具有将初猜解附近划定域进行泛搜能力, 使迭代运算具有强收敛性和平稳性。该方法所定义的迭代因子可通过编程自动调整, 进而可以确定最小迭代次数及此意义下的最优迭代因子。

主题词: 涡轮喷气发动机 数学模型 非线性方程组 优化搜索方法  
分类号: O241.7

## 1 “有限域优化搜索法”思想

求解非线性涡扇发动机数学模型的传统 N-R 方法是以优化迭代方法不断地对猜解进行修正, 使其逼近方程组真解。从理论上讲, 合适的初猜解总是能保证 N-R 求解的迭代运算收敛。但要给出合适的初猜解往往不易, 尤其是在发动机数学模型的求解中, 按经验数据、公式给出的初始猜解并不能保证在飞行包线的每一点都合适。在这种情况下, 如何保证求解迭代运算的收敛性呢? 从根本上说, 如果能用一种非常原始的方法, 将含方程组真解的初猜解邻域逐点遍搜就一定求出真解。遗憾的是, 这样的方法在数学上不能准确描述。然而, 我们可以把这样遍搜的思想与 N-R 法相结合, 构成“有限域优化搜索法”。具体实现方法如下: 设有多元非线性方程组  $F(X) = 0$ ,  $F(X) = [f_1(X) \ f_2(X) \ \cdots \ f_n(X)]^T$ ,  $X = [x_1 \ x_2 \ \cdots \ x_n]^T$ 。

(1) 给出方程组解的初猜解向量  $X_0 = [x_{10} \ x_{20} \ \cdots \ x_{n0}]^T$ 。选择猜解增量调整因子  $ACF$ , 猜解增量限定因子  $ACR1$ , 猜解限定因子  $ACR2$ 。给出迭代求解运算收敛的方程残差指标向量:  $E = [\epsilon_1 \ \epsilon_2 \ \cdots \ \epsilon_n]^T$ 。

(2) 用 N-R 方法进行基本的求解运算求出猜解增量  $\Delta X^k = [FX/X]_{\bar{X}^k}^{-1} \cdot F(X_k)$ , ( $k$  为迭代次数)。 $\Delta X_k = [\Delta x_{1k} \ \Delta x_{2k} \ \cdots \ \Delta x_{nk}]^T$ ,  $X_k = [x_{1k} \ x_{2k} \ \cdots \ x_{nk}]^T$ ,  $F(X_k) = [f_1(X_k) \ f_2(X_k) \ \cdots \ f_n(X_k)]^T$ 。

(3) 若  $|f_i(X_k)| < \epsilon_i$ , ( $i = 1, 2, \cdots, n$ ), 那么迭代求解运算结束, 否则进行下一步。

(4) 若  $|\Delta x_{ik}/x_{i0}| < ACR1$  ( $i = 1, 2, \cdots, n$ ), 就进行到第 7 步, 否则进行第 5 步。

(5)  $\Delta x_{ik} = \Delta x_{ik}/ACF$ , ( $i = 1, 2, \cdots, n$ )。

(6) 返回第 4 步。

- (7)  $x_{ik+1} = x_{ik} - \Delta x_{ik}$  ,  $(i = 1, 2, \cdots, n)$ 。
- (8) 若  $|(x_{ik+1} - x_{i0})/x_{i0}| > ACR2$ , 取  $x_{ik+1} = x_{i0} - \Delta x_{ik}(i = 1, 2, \cdots, n)$ , 否则  $x_{ik+1}$  仍为第 7 步中算出的值。
- (9) 返回第二步。
- $ACF$  和  $ACR1$  作用在于通过调整猜解修正增量大小, 从而影响猜解落入收敛区域的快慢;  $ACR2$  作用在于迭代点超出了划定区域时, 按  $x_{ik+1} = x_{i0} - \Delta x_{ik}$  自动回头, 从而使得迭代运算在划定区域内有一定泛搜力。一般地, 考虑到发动机初猜解给定精度, 优化搜索法中 3 个因子的取值范围为  $1.1 < ACF < 10$  (唯特殊情况: 增量很小迭代收敛十分缓慢时  $ACF$  可取小于 1.0),  $1\% < ACR1 < 8\%$ ,  $10\% < ACR2 < 30\%$ 。在实际编程中, 可以按要求的迭代收敛次数来自动调整确定  $ACF$ 、 $ACR1$  和  $ACR2$  的值。由此可确定保证收敛精度下的最少迭代次数及最优迭代因子。

2 有限域优化搜索法在发动机数学模型求解运算中的应用

本文所用模型来自文献 [1] 中的某型涡扇发动机数学模型。稳态模型有 4 个猜值:  $PNF$  (低压转子转速相对于设计点物理转速百分比),  $PNC$  (高压转子转速相对于设计点物理转速百分比),  $XWA22C$  (按风扇出口气流参数计算的换算流量  $\text{kg} \cdot \text{Pa/s} \cdot \sqrt{\text{K}}$ ),  $XWA3C$  (按高压压气机出口气流参数计算的换算流量  $\text{kg} \cdot \text{Pa/s} \cdot \sqrt{\text{K}}$ )。动态模型有 2 个猜值:  $XWA22C$  和  $XWA3C$ 。

表 1 选取了在飞行包线内较有代表意义的 10 个工作点。用传统的 N-R 求第 1 点时, 迭代运算振荡大。  $PNF$  的初值为 100.0, 第 1 次迭代结果为 771.3, 第 2 次迭代结果为 -234.4, 第 3 次迭代结果为 2782.0, …… , 迭代中, 初猜解的精度意义完全丧失, 迭代发散。用优化搜索法求解时, 由于对每一步的猜解修正量进行了限制, 迭代过程中固守了初猜解的精度意义, 所以迭代运算过程平稳:  $PNF$  第 1 次迭代结果为 100.7, 第 2 次迭代结果为 100.5, 第 3 次迭代结果为 100.6, …… , 迭代至第 8 次稳态点被求出。

表 1 在飞行包线内的几个发动机稳态工作点

| 序号 | 飞行高度(km) | 飞行马赫数 | 尾喷面积(m <sup>2</sup> ) | 主燃烧室供油量(kg/s) |
|----|----------|-------|-----------------------|---------------|
| 1  | 0.0      | 0.0   | 0.1928686             | 1.2004488     |
| 2  | 0.0      | 0.0   | 0.3241149             | 0.0841928     |
| 3  | 4.0      | 0.0   | 0.3241149             | 0.0841948     |
| 4  | 0.3      | 0.65  | 0.1928686             | 0.6210877     |
| 5  | 20.0     | 1.231 | 0.1928686             | 0.0859497     |
| 6  | 20.0     | 0.87  | 0.1928686             | 0.0821000     |
| 7  | 0.0      | 0.0   | 0.1928686             | 0.2727980     |
| 8  | 0.0      | 0.0   | 0.3241149             | 0.1950390     |
| 9  | 0.0      | 0.0   | 0.3241149             | 0.0907080     |
| 10 | 0.0      | 0.0   | 0.3241149             | 0.097504      |

用传统 N-R 法求解第 1、2、9、10 点迭代运算不收敛, 如表 2。

表 2 传统 N-R 法求解发动机数学模型结果

| 序号   | 1   | 2   | 3  | 4  | 5  | 6  | 7  | 8  | 9   | 10  |
|------|-----|-----|----|----|----|----|----|----|-----|-----|
| 迭代次数 | 15  | 2   | 3  | 2  | 2  | 4  | 3  | 4  | 1   | 5   |
| 是否收敛 | 不收敛 | 不收敛 | 收敛 | 收敛 | 收敛 | 收敛 | 收敛 | 收敛 | 不收敛 | 不收敛 |

注: 表中  $ACF$  均为 1.0

表 3 优化搜索法求解发动机数学模型结果

| 序号     | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|--------|------|------|------|------|------|------|------|------|------|------|
| $ACR1$ | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 |
| $ACF$  | 1.10 | 1.40 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.70 | 1.20 |
| 迭代次数   | 7    | 10   | 4    | 2    | 2    | 6    | 6    | 7    | 8    | 8    |
| 是否收敛   | 收敛   | 收敛   | 收敛   | 收敛   | 收敛   | 收敛   | 收敛   | 收敛   | 收敛   | 收敛   |

注: 表中  $ACR2$  均为 0.30

用有限域优化搜索法求解时, 限定迭代次数不超过 10 次, 自动整定因子  $ACF$ 、 $ACR1$  和  $ACR2$ 。求解结果列于表 3。不断减少迭代限定次数, 可确定保证  $10^{-5}$  收敛精度下, 求解第 1 点的最少迭代次数为 3 次, 求解第 9 点的最少迭代次数为 5 次, …… , 从而得到求解这些点的最优迭代因子。计算发动机从慢车至最大状态的动态响应过程为例。在这个过程中,  $WFB$  在 4 秒内从慢车供油量 0.0841928 按线性增量  $\Delta WFB = (1.2004488 - 0.0841928) / 4.0 \cdot t$  增至最大状态供油量 1.2004488; 当  $PNC < 73\%$  时,  $A_8 = 0.3241149$ , 当  $PNC \geq 73\%$  时, 按  $\Delta A_8 = -(0.3241149 - 0.1928686) / 2.0 \cdot t$  在两秒内收至小喷口面积 0.1928686。 $PNF$  和  $PNC$  响应曲线见图 1。从计算过程可以看到, 动态过程计算十分平稳。

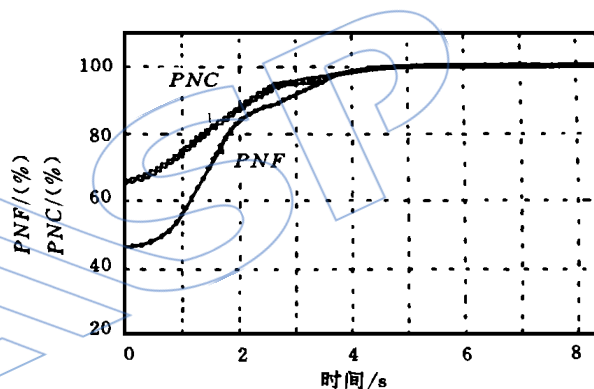


图 1 慢车至最大状态的加速过程

### 3 结 论

有限域优化搜索法把遍搜思想和传统 N-R 方法相结合, 既具有 N-R 求解的特色, 又使得迭代求解运算具有了有限定域内泛搜的能力, 从而使得初猜解的给定要求降低; 而且迭代中固守了初猜解的精度意义, 使得用它来求解象发动机数学模型这样的非线性方程组时, 比传统的 N-R 方法收敛性强、平稳性好。

### 参 考 文 献

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## 3D TURBULENT FLOW AT COMPRESSOR ROTOR EXIT UNDER DIFFERENT FLOW CONDITIONS

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### ABSTRACT

Detailed measurements of three-dimensional turbulent flow field at the exit of an axial flow single-stage compressor rotor were made under different conditions. The flow structures were surveyed with both hot-wire probe and high frequency pressure probe. Under the design condition, substantial blockage, loss and turbulence mainly emerge in the tip mid-passage, in the wake and the corner between the suction surface and the hub, and the tip leakage vortex exists in the mid-passage at the exit. When the mass flow reduces, the flow becomes more turbulent. As a result, flow mixing is more, and the leakage vortex is far away from the suction surface, that leads to more substantial blockage, loss and turbulence in the tip region. Under the near stall condition, the radial migration of the blade suction surface boundary layer and the tangential migration of the endwall flow play a more important role in the flow field of the passage.

**Key Words** Compressors Rotors Vortex Turbulent flow

## APPLICATION OF LIFTING SURFACE METHOD TO RAM AIR TURBINE PERFORMANCE CALCULATION

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### ABSTRACT

The modern propeller lifting surface method has been applied to calculating the performance of Ram Air Turbine, and a single layer vortex model is adapted to the blade, the wake is modeled to rigid cylinder wake coincided with Goldstein hypothesis. A numerical calculation for a model of Ram Air Turbine shows a good agreement of the coefficient of wind power utilization with its experimental data at both small and large loading. The distributions of induced velocities and circulation are reasonable.

**Key Words** Ram air turbines Lifting Surface performance analysis

## A LIMITED DOMAIN SEARCHING METHOD FOR SOLUTION OF NONLINEAR MATHEMATICAL MODEL FOR TURBOFAN ENGINE

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### ABSTRACT

An algorithm is developed based on the N - R (Newton - Raphson) iteration method to solve the nonlinear mathematical model of turbofan engine. It enhances the ability to search for the nonlinear equation solutions within a limited domain, which is designated around the initial guess of solution and covers the genuine solution. Three iteration factors defined in this algorithm can be automatically readjusted by programming. Accordingly, the least times of iteration and corresponding optimum iteration factors

can be found out from the program.

**Key Words** Turbofan engines Mathematical simulation Searching method

## NUMERICAL PREDICTION OF TWO- DIMENSIONAL TRANSONIC TURBULENT FLOW IN A DIFFUSER

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**ABSTRACT** Two- dimensional transonic flow in a diffuser is numerically investigated by solving 2D time- dependent Reynolds- averaged compressible Navier- Stokes equations. The numerical method is based on Van- Leer's flux splitting cell- centered finite volume scheme. Baldwin- Lomax algebraic turbulence model and Johnson- King non- equilibrium turbulence model are tested to simulate largely separated flow. Comparison of the computational results with corresponding experimental data shows that the predicted wall static pressure distributions and shock strength by both two turbulence models agree well with the experimental data, but Johnson- King model does better than Baldwin- Lomax model in the prediction of separation reattachment point.

**Key Words** Shock wave Turbulence Numerical computation

## 3D VISCOUS FLOW COMPUTATION WITH ARBITRARY LAGRANGIAN- EULERIAN METHOD

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**ABSTRACT** Arbitrary Lagrangian- Eulerian schemes are deduced for computation of three dimensional viscous flow in rotating impeller channel. The convection effect is simulated by the pseudo- moving of grids. Viscous sub- cycle time step and Lagrangian sub- cycle time step are adopted to accelerate convergence. The upwind scheme is improved into part- donated scheme to diminish diffusion. Flowrate restriction is used to control dissipative truncation error. A short wave damp is introduced to reduce the computational corrugation. Sub- Grid- Scale(SGS) turbulent model is adopted to simulate turbulent flow. Eckardt's experimental results on centrifugal impeller have verified effectiveness of the method.

**Key Words** Viscous flow Three dimensional flow Numerical computation

## OPTIMIZATIONAL DESIGN FOR TRANSONIC TURBINE CASCADE

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**ABSTRACT** A design method of aerodynamic optimization is presented for transonic turbine cascades. The method couples Genetic Algorithms (GAs) to a inviscid flow field Euler solver and the boundary layer calculation. The objective function of the transonic turbine cascade is the minimum of loss factor. The multinomial coefficients of blade geometry are taken as design variables. RA cascade and Beizhong cascade have been used as the exam-